Misleading Coupling Unification and Lifshitz Type Gauge Theory

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We propose a new grand unification scenario for ensuring proton stability. Our scenario is based on the idea that the proton decay problem is an artificial one, which is caused from the identification of the gauge coupling unification scale with the grand unification scale or the grand unified symmetry breaking scale. We discuss a Lifshitz type gauge theory as a candidate to realize our scenario.

§1. Introduction

Grand unification is an attractive idea and enables the unification of forces and the (partial) unification of quarks and leptons in each family.[1] The introduction of supersymmetry (SUSY) increases the reality of grand unification. In the minimal supersymmetric standard model (MSSM), the gauge couplings meet at $M_X = 2.1 \times 10^{16}$ GeV if the superpartners and Higgs particles exist below or around $O(1)$TeV.[2],[**] It is natural to guess that the grand unification occurs at M_X and physics above M_X can be described as a supersymmetric grand unified theory (SUSY GUT). This scenario is very attractive, but, in general, it suffers from problems related to Higgs multiplets. The typical one is the proton decay problem[4] in the minimal SUSY GUT.[5]

The proton decay problem comes from a significant contribution from the dimension 5 operators. Stronger constraints on the colored Higgs mass M_{H_C} and the sfermion mass $m_{\tilde{f}}$ have been obtained (e.g. $M_{H_C} > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV) from analysis including a Higgsino dressing diagram with right-handed matter fields in the minimal SUSY $SU(5)$ GUT.[6] Extra particles such as X, Y gauge bosons, the colored Higgs bosons and their superpartners are expected to acquire heavy masses of $O(M_U)$ after the breakdown of grand unified symmetry. Here M_U is the grand unification scale or the grand unified symmetry breaking scale. The constraints contradict with the relation $M_{H_C} = O(M_U)$ if M_U is identified with the gauge coupling unification scale M_X. From this observation, we come across the conjecture that *the proton stability can be realized to be compatible with the grand unification, if M_X is not directly related to M_U and the order of M_U is bigger than that of M_X.* It might lead to the idea that gauge couplings agree with accidentally at M_X.

In this paper, we propose a new grand unification scenario for ensuring proton stability on the basis of the above conjecture and the standpoint that *the scale M_X has a physical meaning beyond the fact that the agreement of gauge couplings occurs accidentally there.* We discuss a Lifshitz type gauge theory as a candidate to realize

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[**] The gauge coupling unification based on SUSY was predicted in Refs. 3).

our scenario.

This paper is organized as follows. In the next section, we elaborate our scenario. In §3, we discuss a candidate to realize our scenario. In §4, we present conclusions and discussion. In Appendix A, we study radiative corrections using a Lifshitz type abelian gauge theory.

§2.   Our scenario

The proton stability can be threatened through contributions from the dimension 5 operators including the colored Higgsinos in the minimal SUSY $SU(5)$ GUT. Stronger constraints on the colored Higgs mass M_{H_C} and the sfermion mass $m_{\tilde{f}}$ have been obtained (e.g. $M_{H_C} > 6.5 \times 10^{16}$GeV for $m_{\tilde{f}} < 1$TeV) from analysis including a Higgsino dressing diagram with right-handed matter fields.[6],[*] Extra particles such as X, Y gauge bosons, the colored Higgs bosons and their superpartners usually acquire heavy masses of $O(M_U)$ after the breakdown of grand unified symmetry at M_U. The root of proton decay problem stems from the relation $M_{H_C} = O(M_U)$ with the identification between the grand unified symmetry breaking scale M_U and the gauge coupling unification scale $M_X = 2.1 \times 10^{16}$GeV. Conventionally, one tackles the problem through the extension of the model leaving the identification untouched.
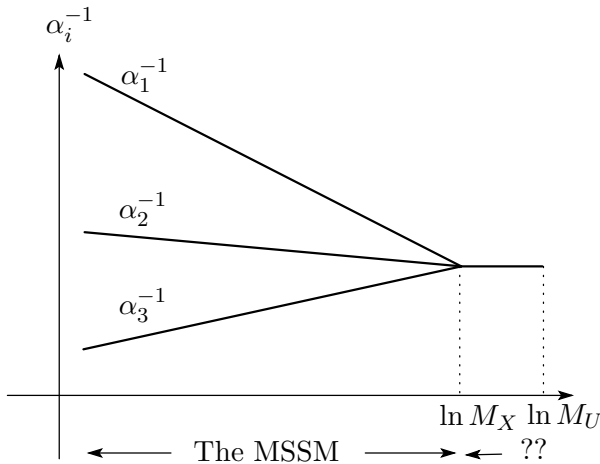

Fig. 1.   Running of α_i^{-1}.

We explore a new possibility of constructing a problem-free model. The underlying idea of our scenario is that *the gauge coupling unification at M_X provides misleading information that the gauge symmetries are also unified at M_X. The proton stability can be realized to be compatible with the grand unification, if the grand unified symmetry breaking scale is not M_X but a bigger than that. The scale M_X has a physical meaning beyond the fact that the agreement of gauge couplings occurs accidentally there.* The outline of our scenario is as follows. A GUT describes physics above M_U, which is bigger than M_X to suppress the prodon decay processes sufficiently. After the breakdown of grand unified symmetry, the running of standard model (SM) gauge couplings g_i, in general, differs. In this case, it is necessary to explain why g_i meet again at M_X. It might occur due to a decoupling of extra particles. In this paper, we explore a more exotic scenario that g_i run (or

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stand) together from M_U to M_X resulted from a specific dynamics.[*] Then the stucture of theory changes around M_X as a result of a phase transition irrelevant to the grand unification and the different running of g_i starts there. A typical running of gauge couplings is depicted in Fig. 1. The α_i ($i = 1, 2, 3$) are structure constants constructed from g_i. Through two kinds of phase transitions, the MSSM will be derived in the following,

$$\int dt d^D \boldsymbol{x} \mathcal{L}_U \xrightarrow{M_U} \int dt d^{\tilde{D}} \boldsymbol{x} \mathcal{L}_I \xrightarrow{M_X} \int d^4 x \mathcal{L}_{\mathrm{MSSM}} \ , \tag{2·1}$$

where $\mathcal{L}_U$, $\mathcal{L}_I$ and $\mathcal{L}_{\mathrm{MSSM}}$ are the Lagrangian density of GUT, an intermediate theory and the MSSM and we have left the possibility that the structure of space-time is also changed.

For our scenario, it is natural to ask the following questions:
1. What kind of GUT describes physics above M_U?
2. How is the grand unified symmetry broken down to a smaller one at M_U?
3. What kind of theory describes physics at the region between M_U and M_X?
4. What kind of dynamics makes g_i to run (or stand) together from M_U to M_X?
5. What kind of phase transition occurs to derive the MSSM around M_X?
Unfortunately, we presently have no definite answers to these questions, although we come some conjectures to mind. Let us look for possible answers by taking the fourth question as a clue. The simplest possibility is that any gauge couplings do not (almost) run between M_U and M_X, i.e., the beta functions of g_i (almost) vanish. We expect that it can stem from a specific superrenormalizable interactions. Then a phase transition occurs and a theory with superrenormalizable interactions changes into that with renormalizable interactions. We take up challenges for this possibility inspired by models with a Lifshitz type fixed point.[9),10)] For the first and the third questions, we expect that the theory has a Lifshitz type fixed point with $z > 1$ above M_X, it has a grand unified symmetry above M_U and the SM symmetry below M_U, and it changes into a renormalizable theory with $z = 1$ like the MSSM around M_X. In this case, the Poincare invariance and SUSY are regarded as accidental symmetries which emerge in the lower energy scale less than M_X or in the infrared (IR) region.[**]

§3. A candidate

We expect that a specific gauge theory possesses a fixed point with anisotropic scaling characterized by dynamical critical exponent $z > 1$ and realizes our scenario,

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[*] The idea that the proton decay rate is suppressed by the increase of the unification scale has been proposed with interesting examples realized by the existence of extra vector-like split multiples.[8)] In particular, the pattern of running of gauge couplings in "fake unification" is same as that in our scenario, but the causes are different, i.e., the runnings are deeply related to the existence of extra particles in theirs and a specific dynamics in ours.

[**] In Refs. 11), properties and renormalizability for quantum field theories with Lorentz symmetry breaking terms have been studied intensively under "weighted power counting". Furthermore the extensions of the SM have been proposed in this framework.[12)]

but we need a more careful consideration to construct a complete theory. Here, we present a candidate and discuss its features.

3.1. *Preparations*

Space-time is assumed to be factorized into a product of D-dimensional Euclidean space $\boldsymbol{R}^D$ and a time $\boldsymbol{R}$, whose coordinates are denoted by x^i ($i = 1, \cdots, D$) (or $\boldsymbol{x}$) and t, respectively. In this paper, we study mostly the case with $D = 3$. The dimensions of x^i and t are defined as

$$[x^i] = -1 \ , \quad [t] = -z \ , \tag{3.1}$$

where z is the dynamical critical exponent which characterizes anisotropic scaling $x^i \to bx^i$ and $t \to b^z t$ at the fixed point. The system does not possess the relativistic invariance for $z \neq 1$ but spatial rotational invariance and translational invariance. The Lorentz invariance is expected to emerge after the transition from $z \neq 1$ to $z = 1$.

Let us first consider a simple model on $\boldsymbol{R}^3 \times \boldsymbol{R}$ with a complex scalar field ϕ and two kinds of spinor fields φ and η described by the action:

$$S_0 = \int dt d^3 \boldsymbol{x} \left[\left| \frac{\partial \phi}{\partial t} \right|^2 - \frac{1}{\kappa^2} \left| \frac{\partial^2 \phi}{\partial x^i \partial x^i} \right|^2 \right.$$
$$\left. + \varphi^\dagger i \frac{\partial}{\partial t} \varphi + \eta^\dagger i \frac{\partial}{\partial t} \eta + \frac{1}{\xi^2} \varphi^\dagger \frac{\partial^2}{\partial x^i \partial x^i} \eta + \frac{1}{\xi^2} \eta^\dagger \frac{\partial^2}{\partial x^i \partial x^i} \varphi \right] \ . \tag{3.2}$$

Both φ and η are 2-component spinors defined on $\boldsymbol{R}^3$ and they transform as

$$\varphi(x) \to \varphi' \to \varphi'(x') = S(O)\varphi(x) \ , \quad \eta(x) \to \eta' \to \eta'(x') = S(O)\eta(x) \ , \tag{3.3}$$

$$S(O) \equiv e^{-\frac{i}{4}\omega_{ij}\sigma^{ij}} \ , \quad \sigma^{ij} \equiv \frac{i}{2} \left(\sigma^i \sigma^j - \sigma^j \sigma^i \right) \ , \tag{3.4}$$

under the 3-dimensional rotation $x^i \to x'^i = O^i_j x^j$. Here, σ^is are Pauli matrices, ω_{ij} are parameters related to rotation angles θ^i such as $\omega_{ij} = -\varepsilon_{ijk}\theta^k$ and O^i_j is the 3×3 orthogonal matrix given by $O^i_j = (e^\omega)^i_j$. The $S(O)$ satisfies the following relations:

$$S^\dagger(O)\sigma^i S(O) = O^i_j \sigma^j \ , \quad S^\dagger(O)S(O) = I \ , \tag{3.5}$$

where I is the 2×2 unit matrix. The terms such as $\varphi^\dagger \partial_i \partial_i \varphi$ and $\eta^\dagger \partial_i \partial_i \eta$ are also invariant under the spatial rotation, but we assume that they are absent. Then (3.2) is rewritten as

$$S_0 = \int dt d^3 \boldsymbol{x} \left[\left| \frac{\partial \phi}{\partial t} \right|^2 - \frac{1}{\kappa^2} \left| \frac{\partial^2 \phi}{\partial x_i \partial x_i} \right|^2 + \bar{\psi} i \gamma^0 \frac{\partial}{\partial t} \psi + \frac{1}{\xi^2} \bar{\psi} \frac{\partial^2}{\partial x_i \partial x_i} \psi \right] \ , \tag{3.6}$$

where we define a 4-component spinor field as $\psi \equiv (\varphi, \eta)^t$, $\bar{\psi} \equiv \psi^\dagger \gamma^0$ and γ^0 corresponds to a time component of the gamma matrices given by

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \ . \tag{3.7}$$

The engineering dimensions of fields (ϕ, ψ) and couplings (κ^2, ξ^2) are given by

$$[\phi] = \frac{3-z}{2} \ , \quad [\psi] = \frac{3}{2} \ , \quad [\kappa^2] = 4 - 2z \ , \quad [\xi^2] = 2 - z \ . \tag{3.8}$$

The system has a free-field fixed point with $z = 2$.

Next let us adopt the gauge principle. The gauge fields (A_t, A_i) are introduced and the derivatives $(\partial_t \equiv \partial/\partial t, \partial_i \equiv \partial/\partial x^i)$ are replaced into the covariant ones (D_t, D_i) such as

$$\partial_t \Rightarrow D_t \equiv \partial_t + igA_t \ , \quad \partial_i \Rightarrow D_i \equiv \partial_i + igA_i \ , \tag{3.9}$$

where g is a gauge coupling. The engineering dimensions of A_t and A_i are

$$[A_t] = z - [g] \ , \quad [A_i] = 1 - [g] \ . \tag{3.10}$$

Gauge field strengths (F_{ti}, F_{ij}) are constructed from the commutators of covariant derivatives as

$$[D_t, D_i] = igF_{ti} \ , \quad [D_i, D_j] = igF_{ij} \ . \tag{3.11}$$

If the gauge fields are dynamical, the kinetic term of gauge fields should be added. In this way, we obtain the following gauge invariant action:

$$S = \int dt d^3\boldsymbol{x} \left[\mathrm{tr}(F_{ti})^2 - \frac{1}{2\lambda^2}\mathrm{tr}(D_k F_{ij})^2 \right.$$
$$\left. + |D_t\phi|^2 - \frac{1}{\kappa^2}|D_i D_i\phi|^2 + \bar{\psi}i\gamma^0 D_t\psi + \frac{1}{\xi^2}\bar{\psi}D_i D_i\psi \right] \ , \tag{3.12}$$

where tr represents the trace for the gauge generators. If the differential operator $\partial_i\partial_i$ for φ and η in (3.2) is regarded as $(\sigma^i\partial_i)^2$, the last term in (3.12) is replaced by $-\xi^{-2}\bar{\psi}(\gamma^i D_i)^2\psi$. Here, γ^is correspond to space components of gamma matrices given by

$$\gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \tag{3.13}$$

and satisfy the relation $\gamma^i\gamma^j + \gamma^j\gamma^i = -2\delta^{ij}$. We add $\mathrm{tr}(D_k F_{ij})^2$ as the 'potential term' for A_i to the action S with an extended model with a higher spatial derivative in mind. The 'potential term' means terms other than the kinetic term including time derivatives. If we use the detailed balance condition, the term proportional to $\mathrm{tr}(D_i F_{ik} D_j F_{jk})$ is introduced. Here, the detailed balance condition means that the action (S_{D+1}) in $D + 1$ dimensions is constructed from the action (S_D) in D dimensions through a particular procedure using functional derivatives. For example, in a model with a real scalar field Φ, the action is constructed as

$$S_{D+1} = \int dt d^D\boldsymbol{x} \left[\frac{1}{2}\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2\kappa}\frac{\delta S_D}{\delta\Phi}\right)^2 \right] \ , \tag{3.14}$$

up to total derivative term. The dimension of $[\mathrm{tr}(F_{ti})^2]$ is given by $3 + z$. Then the dimensions of (A_t, A_i) and (g, λ^2) are

$$[A_t] = \frac{1+z}{2} \ , \ \ [A_i] = \frac{3-z}{2} \ , \ \ [g] = \frac{z-1}{2} \ , \ \ [\lambda^2] = 4 - 2z \ . \tag{3.15}$$

The effective field theory, in general, contains operators whose dimensions equal to or less than those in S, which are not forbidden by symmetry. We add the following relevant terms directly to S:

$$\Delta S = \int dt d^3\boldsymbol{x} \left[-\frac{\tilde{c}_A^{2z-2}}{2} \mathrm{tr}(F_{ij})^2 - \tilde{c}_\phi^{2z-2}|D_i\phi|^2 + \tilde{c}_\psi^{z-1}\bar{\psi}i\gamma^i D_i\psi + \cdots \right] \ , \tag{3.16}$$

where $\tilde{c}_A$, $\tilde{c}_\phi$ and $\tilde{c}_\psi$ are parameters with dimension one and the ellipsis stands for terms related to other operators such as Yukawa interactions and self-interactions of scalar fields. The third term in (3.16) is written in terms of 2-component spinors φ and η as

$$\tilde{c}_\psi^{z-1}\bar{\psi}i\gamma^i D_i\psi = \tilde{c}_\psi^{z-1}\left(\varphi^\dagger i\sigma^i D_i\varphi - \eta^\dagger i\sigma^i D_i\eta \right) \ . \tag{3.17}$$

It is straightforward to construct a model with a complex scalar field ϕ, a $2^{[(D+1)/2]}$-component spinor field ψ and gauge fields (A_t, A_i) on $\boldsymbol{R}^D \times \boldsymbol{R}$. The superficial degree of divergence (D_s) is defined by

$$D_s \equiv D + z - \frac{D-z}{2}B - \left(\frac{D+z}{2} - 1\right)B_t - \frac{D}{2}F - \sum_k [g^k]n_k - \cdots \ , \tag{3.18}$$

where B, B_t, F and n_k are numbers of external lines for ϕ and A_i, numbers of external lines for A_t, external lines for ψ and vertices including the k-th power of g. The ellipsis represents contributions from other couplings.

In the case with $z = 2$ and $D = 3$, the dimensions of fields and couplings become

$$[\phi] = \frac{1}{2} \ , \ \ [\psi] = \frac{3}{2} \ , \ \ [A_t] = \frac{3}{2} \ , \ \ [A_i] = \frac{1}{2} \ , \tag{3.19}$$

$$[g] = \frac{1}{2} \ , \ \ [\kappa^2] = [\xi^2] = [\lambda^2] = 0 \ , \tag{3.20}$$

and D_s is given by

$$D_s = 5 - \frac{1}{2}B - \frac{3}{2}B_t - \frac{3}{2}F - \frac{1}{2}\sum_k kn_k \ . \tag{3.21}$$

The theory is superrenormalizable by power counting and infinities appear for a finite number of diagrams if exist. In Appendix A, we study radiative corrections at one-loop level using an abelian gauge theory and find that radiative corrections for gauge coupling do not contain any infinities owing to the gauge invariance. We expect that the same property holds on the Lifshitz type extension of the MSSM and SUSY GUT.

3.2. *A chiral model*

On the construction of GUT or an intermediate theory, a simple group or the SM gauge group should be chosen as a gauge group with suitable particle contents (including chiral fermions). Here, we do not specify the gauge group and particle contents for simplicity. We discuss a model with $z = 3$ on the space-time $\boldsymbol{R}^3 \times \boldsymbol{R}$, including chiral fermions with a higher spatial derivative term. A candidate is give by the action:

$$S_{z=3} = S + \Delta S \ ,$$

$$S = \int dt d^3\boldsymbol{x} \left[\mathrm{tr}(F_{ti})^2 - \frac{1}{2\lambda^2}\mathrm{tr}(D_k D_l F_{ij})^2 + \sum_f \left(\varphi_f^\dagger i D_t \varphi_f - \frac{1}{\xi^2}\varphi_f^\dagger \boldsymbol{D}^2 i\sigma^i D_i \varphi_f \right) \right.$$

$$\left. + \sum_h \left(|D_t \phi_h|^2 - \frac{1}{\kappa^2}\left| \boldsymbol{D}^2 D_i \phi_h \right|^2 \right) \right] \ ,$$

$$\Delta S = \int dt d^3\boldsymbol{x} \left[-\frac{\tilde{c}_A^4}{2}\mathrm{tr}(F_{ij})^2 + \sum_f \tilde{c}_f^2 \varphi_f^\dagger i\sigma^i D_i \varphi_f - \sum_h \tilde{c}_h^4 |D_i \phi_h|^2 + \cdots \right] \ , \quad (3\cdot22)$$

where φ_f are 2-component spinor fields, ϕ_h are scalar fields, $\boldsymbol{D}^2 = D_i D_i$ and the ellipsis stands for terms related to other operators such as Yukawa interactions and self-interactions of scalar fields. We can, in general, add various kinds of gauge invariant operators whose dimension equals to or less than 6. Here, we take simple ones in $S_{z=3}$. For example, we assume that operators such as $\varphi_f^\dagger \varphi_f$ and $\varphi_f^\dagger \boldsymbol{D}^2 \varphi_f$ are absent and do not appear through radiative corrections. If $\varphi_f^\dagger \varphi_f$ exists, the Lorentz invariance does not appear in the IR region. We expect that the above type of action without specific operators is derived from a more fundamental theory.

The engineering dimensions of fields and parameters are given by

$$[A_t] = 2 \ , \quad [A_i] = 0 \ , \quad [\varphi_f] = \frac{3}{2} \ , \quad [\phi_h] = 0 \ , \quad (3\cdot23)$$

$$[g] = 1 \ , \quad [\lambda^2] = [\xi^2] = [\kappa^2] = 0 \ , \quad [\tilde{c}_A] = [\tilde{c}_f] = [\tilde{c}_h] = 1 \quad (3\cdot24)$$

and D_s is given by

$$D_s = 6 - 2B_t - \frac{3}{2}F - \sum_k kn_k \ . \quad (3\cdot25)$$

This theory is also superrenormalizable by power counting. It is important to study the ultraviolet (UV) behavior of our model more carefully.

From a naive dimensional analysis, ΔS dominates against 'potential terms' in S at the energy scale below $\tilde{c}_A$, $\tilde{c}_f$ and $\tilde{c}_h$, and we expect that the relativistic invariance appears there and the transition occurs from the theory with $z = 3$ to that with $z = 1$. In order to become relativistic below M_X, a finetuning among parameters is required such that $\tilde{c}_A = \tilde{c}_f = \tilde{c}_h \equiv M_\ell$ to a high accuracy and the order of M_ℓ is (one or two digit) bigger than that of M_X. We will discuss constraints for these

parameters from experiments in the next subsection. This relation could be realized as renormalization group (RG) invariants.[*] Then $M_\ell^2 t$ is regarded as an ordinary time variable x_0 in the relativity. Hence we expect that each term in ΔS comes from a common origin, $\tilde{c}_A$, $\tilde{c}_f$ and $\tilde{c}_h$ take a common value at the beginning and they do not receive radiative corrections. If the finetuning relation holds on for all particles and the theory has suitable particle contents (gauge bosons, chiral fermions, Higgs bosons and their superpartners) with suitable gauge quantum numbers, the MSSM can be derived as the theory below M_X and the running of gauge couplings in the IR region is understood.

3.3. *Constraints from experiments*

In the Lifshitz type extension of the MSSM with $z > 1$, the dispersion relations for free fields are given by

$$E^2 = \boldsymbol{p}^2 c_k^2 + m_k{}^2 c_k^4 + \frac{\zeta_k}{M_\ell^{2z-2}} \boldsymbol{p}^{2z} c_k^{4-2z} \ , \tag{3.26}$$

where $\boldsymbol{p}^{2z} = (\boldsymbol{p}^2)^z$, $c_k = c(\tilde{c}_k/M_\ell)^{z-1}$ (c is a speed of light), ζ_k are dimensionless parameters and every particle (except gauge bosons related to unbroken gauge symmetries) acquires the mass m_k after the SUSY and/or electroweak symmetry breaking. Here, c_k is an 'own velocity' (a maximal attainable velocity if $\zeta_k > 0$) for each particle labeled by k.

Let us discuss constraints for parameters from experimental data. The Lorentz violating dispersion relations such as (3.26) are tested by the photon emission from a high-energy particle.[15] The process such as $C \to C + \gamma$ is forbidden (at tree level) by the energy-momentum conservation in the case with the Lorentz invariant dispersion relations such as $E^2 = \boldsymbol{p}^2 c^2 + m_C{}^2 c^4$ for a massive charged particle C and $E^2 = \boldsymbol{p}^2 c^2$ for photon γ. It, however, can occur in the presence of Lorentz symmetry breaking terms. We consider the case with the following dispersion relations:

$$E^2 = \boldsymbol{p}^2 c_C^2 + m_C{}^2 c_C^4 + \frac{\zeta_C}{M_\ell^{n-2}} \boldsymbol{p}^n c_C^{4-n} \ , \tag{3.27}$$

$$E^2 = \boldsymbol{p}^2 c_\gamma^2 + \frac{\zeta_\gamma}{M_\ell^{n-2}} \boldsymbol{p}^n c_\gamma^{4-n} \ , \tag{3.28}$$

where $n = 2z$. Using the kinematics, we derive the following relation:

$$\frac{\zeta_\gamma}{M_\ell^{n-2}} \boldsymbol{p}_\gamma^n c_\gamma^{4-n} = \left(\boldsymbol{p}^2 + \boldsymbol{p}'^2\right)\left(c_C^2 - c_\gamma^2\right)$$

$$+ 2\left(m_C{}^2 c_C^4 + \boldsymbol{p} \cdot \boldsymbol{p}' c_\gamma^2 - EE'\right) + \frac{\zeta_C}{M_\ell^{n-2}}(\boldsymbol{p}^n + \boldsymbol{p}'^n)c_C^{4-n} \ , \tag{3.29}$$

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[*] There has been a proposal that the Lorentz invariance appears at an attractive IR fixed point.[13] In Ref. 14), the RG evolution of $\tilde{c}_h$ has been studied for the Lifshitz type scalar field theory with $z = 2$ and $D = 4$ ($D = 10$) and it has been pointed out that a severe fine-tuning in the UV region seems to be inevitable. This result can give a strong constraint on theories with extra dimensions.

where $\boldsymbol{p}_\gamma$ is the momentum of photon, $\boldsymbol{p}$ ($\boldsymbol{p}'$) and E (E') are the momentum and energy of incoming (outgoing) particle C. E and E' satisfy the relation (3·27) and the same type one whose $\boldsymbol{p}$ is replaced by $\boldsymbol{p}'$, respectively. The stability of C at some high-momentum $\boldsymbol{p}_{\mathrm{h}}$ means that the relation (3·29) does not hold on at and below $\boldsymbol{p}_{\mathrm{h}}$ and it leads to the following constraints:

$$\frac{|c_C^2 - c_\gamma^2|}{c_C^2} \lesssim \frac{m_C^2 c_C^2}{\boldsymbol{p}_{\mathrm{h}}^2} \ , \quad \zeta_k \lesssim \frac{m_C^2 M_\ell^{n-2}}{\boldsymbol{p}_{\mathrm{h}}^n} c_k^n \ , \tag{3·30}$$

where k is C or γ. We use the relation $c_C^2 = c_\gamma^2$ up to a very tiny number expected from the first constraint to derive the second one. The most stringent constraints come from the case that C is proton (p^+)[*] and they are given by[**]

$$\frac{|c_p^2 - c_\gamma^2|}{c_p^2} \lesssim \frac{m_p^2 c_p^2}{\boldsymbol{p}_{\mathrm{h}}^2} = O(10^{-22}) \ , \quad \zeta_k \lesssim \frac{m_p^2 M_\ell^{n-2}}{\boldsymbol{p}_{\mathrm{h}}^n} c_k^n = O(10^{6n-34}) \ , \tag{3·31}$$

where we use $\boldsymbol{p}_{\mathrm{h}} = 10^{20}$eV from AGASA date[17] and $m_p = 938$MeV. We take $M_\ell = 10^{17}$GeV. The first constraint requires that $\tilde{c}_p = \tilde{c}_\gamma$ to a high accuracy of order $O(10^{-22} M_\ell)$. From the second one, we find that the magnitude of ζ_k can become $O(1)$ if $n \geq 6$. In the case that C is electron (e^-), constraints are given by

$$\frac{|c_e^2 - c_\gamma^2|}{c_e^2} \lesssim \frac{m_e^2 c_e^2}{\boldsymbol{p}_{\mathrm{h}}^2} = O(10^{-16}) \ , \quad \zeta_k \lesssim \frac{m_e^2 M_\ell^{n-2}}{\boldsymbol{p}_{\mathrm{h}}^n} c_k^n = O(10^{12n-40}/2^{2-n}) \ , \tag{3·32}$$

where we use $\boldsymbol{p}_{\mathrm{h}} = 50$TeV from observations of high-momentum gamma ray originated from the Crab nebula[18] and $m_e = 0.51$MeV. We take $M_\ell = 10^{17}$GeV. The first constraint requires that $\tilde{c}_e = \tilde{c}_\gamma$ to a high accuracy of order $O(10^{-16} M_\ell)$. From the second one, we find that the magnitude of ζ_k can become $O(1)$ if $n \geq 4$.

If the magnitude of Lorentz symmetry breaking term can be within the range of observation concerning photon with an ultra high-momentum in the future, we come across the idea that *light led us to the concept of special relativity, but (ironically) it also might teach us to the violation of relativistic invariance and a new physics beyond the SM or the MSSM.*

§4. Conclusions and discussion

We have proposed a new grand unification scenario for ensuring proton stability on the basis of the conjecture that *the proton stability can be realized to be compatible with the grand unification if M_X is not related to M_U* and the standpoint that *the scale M_X has a physical meaning beyond the fact that the agreement of gauge couplings occurs accidentally there.* In our scenario, M_U is supposed to be bigger than M_X to suppress the prodon decay sufficiently and gauge couplings agree with among them from M_U to M_X by a specific dynamics. The candidate is a Lifshitz type gauge

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[*] Strictly speaking, proton does not belong to the SM particles as an elementary particle and hence ζ_p would be zero in our Lifshitz type model.

[**] Various constraints on maximal attainable velocities for particles have been derived in Ref. 16).

theory with $z > 1$, which possesses specific superrenormalizable interactions. The structure of theory changes around M_X after a phase transition irrelevant to the grand unification and the different running of g_i starts there. The transition occurs from the theory with $z > 1$ to that with $z = 1$, and the ordinary renormalizable MSSM is expected to hold on with the emergence of Poincare invariance and SUSY below M_X.

There are open questions for our scenario and/or model. The most serious problem is a finetuning problem related to the speed of light. The relativistic invariance requires a finetuning among parameters such as $\tilde{c}_A = \tilde{c}_f = \tilde{c}_h \equiv M_\ell$ to a high accuracy for all particles that survive in our low-energy world. This comes from the standpoint that the Lorentz invariance emerges as an accidental symmetry. The relation could be realized as RG invariants. Then the RG invariance (or no running of parameters) would become one of conditions to select a realistic model. We need to study radiative corrections or RG behavior in the space of all the couplings in our model, bearing the finetunig problem in mind. Furthermore M_ℓ might be a fundamental constant in a fundamental theory, e.g. the string scale in string theory. In fact, it is known that the string scale is about 20 times bigger than M_X in the heterotic string theory. Or we might need to reconsider the relation between the kinematics of each particle and the structure of space-time. Other notorious problem is the triplet-doublet Higgs mass splitting problem. There is a possbility that the mass splitting can be realized on the orbifold breaking[19] if a higher-dimensional gauge theory holds on above M_U. It is interesting to study this problem in our framework. It is also important to explore phenomenological implications on the violation of Lorentz invariance at a high-energy scale, considering phenomenological aspect of Hořava-Lifshitz gravity.[*]

Even if our present model do not work for any reason, study of elementary particle physics using Lifshitz type quantum field theories would be attractive and it is worth exploring a high-energy theory on the basis of our proposal. In any event, we would like to believe that nature use the idea of grand unification in order to well-organize physical laws.[**],[***]

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[*] Recently, various aspects of Hořava-Lifshitz gravity have been intensively investigated from its renormalizability to cosmological implications.[20]

[**] Recently, unconventional idea that grand unification is realized in unphysical world has been proposed.[21]

[***] On the other hand, there is an interesting proposal that the merging of gauge couplings can come from not the grand unification but the property of an underlying fermionic vacuum and there is no danger of proton decay.[22]

Appendix A
—— *Radiative Corrections in Lifshitz Type Abelian Gauge Theory* ——

We study radiative corrections at one-loop level using a Lifshitz type abelian gauge theory with $z = 2$ on the space-time $\boldsymbol{R}^3 \times \boldsymbol{R}$. Our players are a $U(1)$ gauge field (A_t, A_i) and a fermion ψ, which is a 4-component spinor (defined on $\boldsymbol{R}^3$) and has a unit $U(1)$ charge e. The action is given by

$$S = \int dt d^3\boldsymbol{x} \left[\frac{1}{2}(F_{ti})^2 - \frac{1}{4}(\partial_k F_{ij})^2 + \bar{\psi} i\gamma^0 D_t \psi + \bar{\psi} D_i D_i \psi \right.$$
$$\left. - \frac{\tilde{c}_A^2}{4}(F_{ij})^2 + \tilde{c}_\psi \bar{\psi} i\gamma^i D_i \psi - \tilde{m}^2 \bar{\psi}\psi \right] , \qquad (A\cdot 1)$$

where $F_{ti} = \partial_t A_i - \partial_i A_t$, $F_{ij} = \partial_i A_j - \partial_j A_i$, $D_t \equiv \partial_t + ieA_t$ and $D_i \equiv \partial_i + ieA_i$. The engineering dimensions of fields and parameters are given by

$$[A_t] = \frac{3}{2} , \quad [A_i] = \frac{1}{2} , \quad [\psi] = \frac{3}{2} , \quad [e] = \frac{1}{2} , \quad [\tilde{c}_A] = [\tilde{c}_\psi] = [\tilde{m}] = 1 . \quad (A\cdot 2)$$

The Feynman rules are given by

$$\frac{i}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{c}_\psi \boldsymbol{\gamma} \cdot \boldsymbol{p} - \tilde{m}^2 + i\varepsilon} \qquad \text{for propagator of } \psi , \qquad (A\cdot 3)$$

$$\frac{-i(\boldsymbol{p}^2 + \tilde{c}_A^2)}{E_p^2 - \boldsymbol{p}^4 - \tilde{c}_A^2 \boldsymbol{p}^2 + i\varepsilon} \qquad \text{for propagator of } A^t , \qquad (A\cdot 4)$$

$$\frac{i}{E_p^2 - \boldsymbol{p}^4 - \tilde{c}_A^2 \boldsymbol{p}^2 + i\varepsilon} \qquad \text{for propagator of } A^i , \qquad (A\cdot 5)$$

$$-ie\gamma^0 \qquad\qquad\qquad \text{for vertex among } \bar{\psi}, \psi \text{ and } A^t , \qquad (A\cdot 6)$$

$$-ie(p_i + p_i' + \tilde{c}_\psi \gamma_i) \qquad \text{for vertex among } \bar{\psi}, \psi \text{ and } A^i , \qquad (A\cdot 7)$$

$$-2ie^2 \delta_{ij} \qquad\qquad\quad \text{for vertex among } \bar{\psi}, \psi, A^i \text{ and } A^j , \quad (A\cdot 8)$$

where $\boldsymbol{p}^4 \equiv (\boldsymbol{p}^2)^2$ and we take $\partial_t A_t + (\nabla^2 - \tilde{c}_A^2)\partial_i A_i = 0$ as the gauge fixing condition to derive the propagators of A_t and A_i.

Now let us study radiative corrections at one-loop level. Here, we do not consider the terms $\dfrac{\tilde{c}_A^2}{4}(F_{ij})^2$ and $\tilde{c}_\psi \bar{\psi} i\gamma^i D_i \psi$ for simplicity.[*] The vacuum polarizations at one-loop level are given by

$$i\Pi_{tt}(q) = -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \text{Tr} \left(\gamma^0 \frac{1}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{m}^2} \right.$$
$$\left. \cdot \gamma^0 \frac{1}{\gamma^0(E_p - E_q) - (\boldsymbol{p} - \boldsymbol{q})^2 - \tilde{m}^2} \right) , \quad (A\cdot 9)$$

$$i\Pi_{it}(q) = -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \text{Tr} \left(\frac{2p_i - q_i}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{m}^2} \right.$$

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[*] The qualitative features are almost same by the introduction of those terms. We will report results of analysis including those terms elsewhere.

$$\cdot \gamma^0 \frac{1}{\gamma^0(E_p - E_q) - (\boldsymbol{p} - \boldsymbol{q})^2 - \tilde{m}^2}\Bigg) \ , \quad \text{(A·10)}$$

$$i\Pi_{ij}(q) = -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \left[\mathrm{Tr} \left(\frac{2p_i - q_i}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{m}^2} \right. \right.$$

$$\left. \cdot \frac{2p_j - q_j}{\gamma^0(E_p - E_q) - (\boldsymbol{p} - \boldsymbol{q})^2 - \tilde{m}^2} \right)$$

$$+ \mathrm{Tr} \left(\frac{2\delta_{ij}}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{m}^2} \right) \Bigg] \ , \quad \text{(A·11)}$$

where $q = (E_q, \boldsymbol{q})$ (E_q and $\boldsymbol{q}$ are energy and momenta for the external gauge boson, respectively) and Tr represents the trace for the spinor indices. $\Pi_{tt}(q)$, $\Pi_{it}(q)$ and $\Pi_{ij}(q)$ seem contain linear, quadratic and tertiary divergent pieces, but they can be removed from the condition of gauge invariance. Physics must not be changed under the gauge transformations $A_t \to A_t + \partial_t \Lambda$ and $A_i \to A_i + \partial_i \Lambda$ ($\varepsilon_t \to \varepsilon_t + \lambda E_q$ and $\varepsilon_i \to \varepsilon_i + \lambda q_i$ in terms of polarization vectors ε_t and ε_i). This requirement leads to the relations $E_q \Pi_{tt} - \sum_i q_i \Pi_{it} = 0$ and $E_q \Pi_{it} - \sum_j q_j \Pi_{ij} = 0$. Using the above expressions (A·9) – (A·11), we derive the following relations:

$$E_q \Pi_{tt} - \sum_i q_i \Pi_{it} = ie^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \mathrm{Tr} \left(\gamma^0 \frac{1}{\gamma^0(E_p - E_q) - (\boldsymbol{p} - \boldsymbol{q})^2 - \tilde{m}^2} \right.$$

$$\left. - \gamma^0 \frac{1}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{m}^2} \right) \ , \quad \text{(A·12)}$$

$$E_q \Pi_{it} - \sum_j q_j \Pi_{ij} = ie^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \mathrm{Tr} \left(\frac{2p_i - q_i}{\gamma^0(E_p - E_q) - (\boldsymbol{p} - \boldsymbol{q})^2 - \tilde{m}^2} \right.$$

$$\left. - \frac{2p_i + q_i}{\gamma^0 E_p - \boldsymbol{p}^2 - \tilde{m}^2} \right) \ . \quad \text{(A·13)}$$

If the integrals were finite, we could make them zero after the change of integration variables in the first terms. Actually we can regularize $\Pi_{tt}(q)$, $\Pi_{it}(q)$ and $\Pi_{ij}(q)$ keeping the gauge invariance and they have the following forms:

$$\Pi_{tt}(q) = \boldsymbol{q}^2 \Pi(q) \ , \quad \Pi_{it}(q) = E_q q_i \Pi(q) \ ,$$
$$\Pi_{ij}(q) = E_q^2 \delta_{ij} \Pi(q) + \left(\boldsymbol{q}^2 \delta_{ij} - q_i q_j \right) \tilde{\Pi}(q) \ , \quad \text{(A·14)}$$

where $\Pi(q)$ is a finite quantity. For example, $i\Pi(0)$ is calculated as

$$i\Pi(0) = -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \mathrm{Tr} \left(\frac{1}{(\gamma^0 E_p - (\boldsymbol{p}^2 + \tilde{m}^2))^3} \right)$$

$$= -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \mathrm{Tr} \left(\frac{(\gamma^0 E_p + (\boldsymbol{p}^2 + \tilde{m}^2))^3}{(E_p^2 - (\boldsymbol{p}^2 + \tilde{m}^2)^2)^3} \right)$$

$$= -4ie^2 \int_{-\infty}^{\infty} \frac{dE d^3\boldsymbol{p}}{(2\pi)^4} \frac{-3E^2(\boldsymbol{p}^2 + \tilde{m}^2) + (\boldsymbol{p}^2 + \tilde{m}^2)^3}{(E^2 + (\boldsymbol{p}^2 + \tilde{m}^2)^2)^3}$$

$$= -4ie^2 \int_{-\infty}^{\infty} \frac{d^3\boldsymbol{p}}{(2\pi)^4} \left(\frac{-3}{8} \frac{\pi}{\boldsymbol{p}^2 + \tilde{m}^2} + \frac{3}{8} \frac{\pi}{\boldsymbol{p}^2 + \tilde{m}^2} \right) = 0 \ , \quad \text{(A·15)}$$

where the Wick rotation is performed ($E_p \to iE$).

In the same way, we can calculate the fermion self-energy and the vertex functions. The expression for the fermion self-energy is given at one-loop level,

$$-i\Sigma(q) = e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{(2\boldsymbol{q}-\boldsymbol{p})^2}{E_p^2 - \boldsymbol{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$+ 6e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{1}{E_p^2 - \boldsymbol{p}^4}$$

$$+ e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{-\boldsymbol{p}^2}{E_p^2 - \boldsymbol{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2} \ , \quad \text{(A·16)}$$

where E_q and q_i are energy and momemta for the external fermion. The expressions for vertex functions among $\bar{\psi}$, ψ and A^t (A^i) are given at one-loop level,

$$-i\Lambda_t(q, q') = e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \gamma^0 \frac{1}{E_p^2 - \boldsymbol{p}^4} \frac{(2\boldsymbol{q}-\boldsymbol{p})\cdot(2\boldsymbol{q}'-\boldsymbol{p})}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$\cdot \frac{1}{\gamma^0(E_{q'} - E_p) - (\boldsymbol{q}'-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$+ e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \gamma^0 \frac{-\boldsymbol{p}^2}{E_p^2 - \boldsymbol{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$\cdot \frac{1}{\gamma^0(E_{q'} - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2} \ , \quad \text{(A·17)}$$

$$-i\Lambda_i(q, q') = e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{1}{E_p^2 - \boldsymbol{p}^4} \frac{(2\boldsymbol{q}-\boldsymbol{p})\cdot(2\boldsymbol{q}'-\boldsymbol{p})}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$\cdot \frac{(q + q' - 2p)_i}{\gamma^0(E_{q'} - E_p) - (\boldsymbol{q}'-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$+ 2e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{(2q - p)_i}{E_p^2 - \boldsymbol{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$+ 2e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{(2q' - p)_i}{E_p^2 - \boldsymbol{p}^4} \frac{1}{\gamma^0(E_{q'} - E_p) - (\boldsymbol{q}'-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$+ e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\boldsymbol{p}}{(2\pi)^4} \frac{-\boldsymbol{p}^2}{E_p^2 - \boldsymbol{p}^4} \frac{(q + q' - 2p)_i}{\gamma^0(E_q - E_p) - (\boldsymbol{q}-\boldsymbol{p})^2 - \tilde{m}^2}$$

$$\cdot \frac{1}{\gamma^0(E_{q'} - E_p) - (\boldsymbol{q}'-\boldsymbol{p})^2 - \tilde{m}^2} \ , \quad \text{(A·18)}$$

where E_q and q_i (E_q' and q_i') are energy and momemta for the incoming (outgoing) fermion. We find that the above vertex functions are UV finite. Furthermore, there exist following identities:

$$\Lambda_t(q, q) = -\frac{\partial}{\partial E_q}\Sigma(q) \ , \quad \Lambda_i(q, q) = \frac{\partial}{\partial q_i}\Sigma(q) \ . \quad \text{(A·19)}$$

Those are counterparts of Ward identity in QED, which guarantee that the fermion self-energy and the three-point vertex functions do not contribute the renormalization of gauge coupling by the cancellation. We find that $D_s < 0$ for diagrams

including loops related to the vertex among $\bar{\psi}$, ψ, A^i and A^j and the proper vertex functions among them are also UV finite.

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